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THE ASYMPTOTIC SOLUTION OF PROBLEMS OF THE ACTION OF A CONCENTRATED FORCE AND A PIECEWISE-CONTINUOUS LOAD ON A TWO-LAYERED STRIP*

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An asymptotic method /1/ is used to solve a new class of elasticity theory problems for a two-layered strip subjected to concentrated and piecewise-continuous normal loads. The problems examined are fundamental, in particular, in analyses of elastic foundations and bases using the model of a compressive layer /2, 3/. A solution is obtained by connecting the asymptotic solution for the strip /4/ with the solution of the Flamant-Boussinesq solution. Fairly simple asymptotic formulas are derived for the desired stress and displacement fields that can find direct application. The approach mentioned can also be extended to laminar plates.

1. Boundary-value problems for a two-layered strip consisting of an isotropic and anisotropic layer under conditions of total contact are examined. It is assumed that the longitudinal edge of the anisotropic part is rigidly clamped while the isotropic part on the analogous edge is subjected to a concentrated force and a piecewise-continuous normal load. We will first examine the case of a concentrated force.

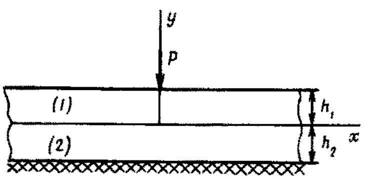

Fig.1

We have the two-layered strip $\Omega = \{x, y\colon -l \leqslant x \leqslant l, -h_2 \leqslant y \leqslant h_1, \quad 2l \gg h_1 + h_2\}$ where the layer $0 \leqslant y \leqslant h_1$ is isotropic and the layer $-h_2 \leqslant y \leqslant 0$ is anisotropic. Let the concentrated normal force

$$\sigma_{yy}(x, h_1) = -P\delta(x), \quad \sigma_{xy}(x, h_1) = 0 \tag{1.1}$$

act on the isotropic layer while the lower face of the anisotropic layer is rigidly clamped

$$u_x(x, -h_2) = u_y(x, -h_2) = 0 \tag{1.2}$$

It is required to determine the state of stress and strain of such a strip (Fig.1) if the complete contact conditions

$$\sigma_{jy}^{(1)}(x, 0) = \sigma_{jy}^{(2)}(x, 0), \quad u_j^{(1)}(x, 0) = u_j^{(2)}(x, 0) \quad (j = x, y) \tag{1.3}$$

are satisfied on the layer line of contact $y = 0$, where the superscipt (1) corresponds to the isotropic layer and the subscript (2) to the anisotropic layer. Arbitrary boundary-value conditions of elasticity theory can be given on the side faces $x = \pm l$. They do not directly influence the procedure for determining the internal solution in the problems under consideration, although the boundary layer is due to them /4, 5/.

We will seek the solution of problem (1.1) and (1.2) in the form

$$Q = \begin{cases} Q_1^{(a)} + Q^{(F)}, & 0 \leqslant y \leqslant h_1 \\ Q_2^{(a)}, & -h_2 \leqslant y \leqslant 0 \end{cases} \tag{1.4}$$

where $Q_1^{(a)}$ and $Q_2^{(a)}$ are general integrals of the asymptotic solutions of mixed boundary-value problems for the strips $\Omega_1 = \{x, y\colon -l \leqslant x \leqslant l, \ 0 \leqslant y \leqslant h_1\}$ and $\Omega_2 = \{x, y\colon -l \leqslant x \leqslant l, -h_2 \leqslant y \leqslant 0, \ l \gg \max(h_1, h_2)\}$, that have the form /4, 6/

$$Q_i^{(a)} = \sum_{(s)} \varepsilon^{\varkappa+s} Q^{(i,s)}, \quad \varkappa_u = 0, \quad \varkappa_\sigma = -1 \tag{1.5}$$

$$\sigma_{xy}^{(i,s)} = \sigma_{xy0}^{(i,s)}(\xi) + \sigma_{xy*}^{(i,s)}(\xi, \zeta)$$

$$\sigma_{yy}^{(i,s)} = \sigma_{yy0}^{(i,s)}(\xi) + \sigma_{yy*}^{(i,s)}(\xi, \zeta)$$

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$$\sigma_{xx}^{(i,s)} = -(a_{12}^{(i)}\sigma_{yy0}^{(i,s)} + a_{16}^{(i)}\sigma_{xy0}^{(i,s)})/a_{11}^{(i)} + \sigma_{xx*}^{(i,s)}(\xi,\zeta)$$
$$u^{(i,s)} = \zeta(A_{26}^{(i)}\sigma_{yy0}^{(i,s)} + A_{22}^{(i)}\sigma_{xy0}^{(i,s)}) + u_0^{(i,s)}(\xi) + u_*^{(i,s)}(\xi,\zeta)$$
$$v^{(i,s)} = \zeta(A_{66}^{(i)}\sigma_{yy0}^{(i,s)} + A_{26}^{(i)}\sigma_{xy0}^{(i,s)}) + v_0^{(i,s)}(\xi) + v_*^{(i,s)}(\xi,\zeta)$$
$$\sigma_{xy*}^{(i,s)} = -\int_0^{\xi}\frac{\partial\sigma_{xx}^{(i,s-1)}}{\partial\xi}d\zeta, \quad \sigma_{yy*}^{(i,s)} = -\int_0^{\xi}\frac{\partial\sigma_{xy}^{(i,s-1)}}{\partial\xi}d\zeta$$
$$\sigma_{xx*}^{(i,s)} = -\frac{1}{a_{11}^{(i)}}(a_{12}^{(i)}\sigma_{yy*}^{(i,s)} + a_{16}^{(i)}\sigma_{xy*}^{(i,s)}) + \frac{1}{a_{11}^{(i)}}\frac{\partial u^{(i,s-1)}}{\partial\xi}$$
$$u_*^{(i,s)} = \int_0^{\xi}\left(a_{16}^{(i)}\sigma_{xx*}^{(i,s)} + a_{26}^{(i)}\sigma_{yy*}^{(i,s)} + a_{66}^{(i)}\sigma_{xy*}^{(i,s)} - \frac{\partial v^{(i,s-1)}}{\partial\xi}\right)d\zeta$$
$$v_*^{(i,s)} = \int_0^{\xi}(a_{12}^{(i)}\sigma_{xx*}^{(i,s)} + a_{22}^{(i)}\sigma_{yy*}^{(i,s)} + a_{26}^{(i)}\sigma_{xy*}^{(i,s)})d\zeta$$
$$\xi = x/l, \quad \zeta = y/h_2 = yl\varepsilon^{-1}, \quad \varepsilon = h_2/l \quad (h_2 > h_1)$$
$$u^{(i)} = u_x^{(i)}/l, \quad v^{(i)} = u_y^{(i)}/l, \quad i = 1, 2$$
$$A_{66}^{(i)} = (a_{11}^{(i)}a_{22}^{(i)} - a_{12}^{(i)2})/a_{11}^{(i)}, \quad A_{22}^{(i)} = (a_{11}^{(i)}a_{66}^{(i)} - a_{16}^{(i)2})/a_{11}^{(i)}$$
$$A_{26}^{(i)} = (a_{11}^{(i)}a_{26}^{(i)} - a_{12}^{(i)}a_{16}^{(i)})/a_{11}^{(i)}$$

The solution (1.5) contains four arbitrary integration functions $\sigma_{xy0}^{(i,s)}$, $\sigma_{yy0}^{(i,s)}$, $u_0^{(i,s)}, v_0^{(i,s)}$; for each (i) $Q^{(F)}$,that satisfies the conditions

$$\sigma_{xy}^{(F)}(x, h_1) = 0, \quad \sigma_{yy}^{(F)}(x, h_1) = -P\delta(x) \tag{1.6}$$

is the solution of the Flamant-Boussinesq problem for a half-plane $\Omega = \{x, y: \ -\infty < x < \infty, -\infty < y \leqslant h_1\}$, that for a piecewise-continuous load has the form /7/

$$\sigma_{xy}^{(F)} = \frac{2}{\pi}I_{12}, \quad \sigma_{yy}^{(F)} = \frac{2}{\pi}I_{03}, \quad \sigma_{xx}^{(F)} = \frac{2}{\pi}I_{21} \tag{1.7}$$
$$u_x^{(F)} = \frac{(1+\nu)(1-2\nu)}{\pi E}\int_a^b p(\xi)\operatorname{arctg}\frac{x-\xi}{y-h_1}d\xi - \frac{1+\nu}{\pi E}I_{11}$$
$$u_y^{(F)} = \frac{2(1-\nu^2)}{\pi E}\int_a^b p(\xi)\ln r(\xi, y)\,d\xi + \frac{1+\nu}{\pi E}I_{20}$$
$$r = r(\xi, y) = \sqrt{(x-\xi)^2 + (y-h_1)^2}$$
$$I_{\alpha\beta} = \int_a^b p(\xi)\frac{(x-\xi)^{\alpha}(y-h_1)^{\beta}}{r^{\gamma}}d\xi$$

where $\gamma = 4$ for $\alpha+\beta = 3$ and $\gamma = 2$ for $\alpha+\beta = 2$.

Satisfying the contact conditions (1.3) and the boundary conditions (1.1) and (1.2) on $y = 0$ we obtain

$$u_0^{(2,s)} = u_0^{(1,s)} + u_0^{(F,s)}(0), \ v_0^{(2,s)} = v_0^{(1,s)} + v_0^{(F,s)}(0) \tag{1.8}$$
$$\sigma_{xy0}^{(2,s)} = \sigma_{xy0}^{(1,s)} + \sigma_{xy}^{(F,s)}(0), \quad \sigma_{yy0}^{(2,s)} = \sigma_{yy0}^{(1,s)} + \sigma_{yy}^{(F,s)}(0)$$
$$\sigma_{xy0}^{(1,s)} = -\sigma_{xy*}^{(1,s)}(\zeta_0), \quad \sigma_{yy0}^{(1,s)} = -\sigma_{yy*}^{(1,s)}(\zeta_0), \quad \zeta_0 = h_1/h_2$$
$$u_0^{(2,s)} = (A_{26}^{(2)}\sigma_{yy0}^{(2,s)} + A_{22}^{(2)}\sigma_{xy0}^{(2,s)}) - u_*^{(2,s)} \quad (\zeta = -1)$$
$$v_0^{(2,s)} = (A_{66}^{(2)}\sigma_{yy0}^{(2,s)} + A_{26}^{(2)}\sigma_{xy0}^{(2,s)}) - v_*^{(2,s)} \quad (\zeta = -1)$$

Hence, the integration functions

$$\sigma_{xy0}^{(1,s)} = -\sigma_{xy*}^{(1,s)}(\zeta_0), \quad \sigma_{yy0}^{(1,s)} = -\sigma_{yy*}^{(1,s)}(\zeta_0) \tag{1.9}$$
$$\sigma_{xy0}^{(2,s)} = -\sigma_{xy*}^{(1,s)}(\zeta_0) + \sigma_{xy}^{(F,s)}(0), \quad \sigma_{yy0}^{(2,s)} = -\sigma_{yy*}^{(1,s)}(\zeta_0) + \sigma_{yy}^{(F,s)}(0)$$
$$u_0^{(2,s)} = A_{26}^{(2)}(\sigma_{yy}^{(F,s)}(0) - \sigma_{yy*}^{(1,s)}(\zeta_0)) + A_{22}^{(2)}(\sigma_{xy}^{(F,s)}(0) - \sigma_{xy*}^{(1,s)}(\zeta_0)) - u_*^{(2,s)}(\zeta = -1)$$
$$v_0^{(2,s)} = A_{66}^{(2)}(\sigma_{yy}^{(F,s)}(0) - \sigma_{yy*}^{(1,s)}(\zeta_0)) + A_{26}^{(2)}(\sigma_{xy}^{(F,s)}(0) - \sigma_{xy*}^{(1,s)}(\zeta_0)) - v_*^{(2,s)}(\zeta = -1)$$
$$u_0^{(1,s)} = u_0^{(2,s)} - u_0^{(F,s)}(0), \qquad v_0^{(1,s)} = v_0^{(2,s)} - v_0^{(F,s)}(0)$$

are uniquely defined, where $\sigma_{xy}^{(F,s)}(0) = \sigma_{yy}^{(F,s)}(0) = u_0^{(F,s)}(0) = v_0^{(F,s)}(0) = 0$ for $s \neq 0$ while $\sigma_{xy}^{(F,0)}(0) = \sigma_{xy}^{(F)}(x = l\xi, y = 0)$, $\sigma_{yy}^{(F,0)}(0) = \sigma_{yy}^{(F)}(x = l\xi, y = 0)$ $u_0^{(F,0)}(0) = l^{-1}u_x^{(F)}(x = l\xi, y = 0)$, $v_0^{(F,0)}(0) = l^{-1}u_y^{(F)}(x = l\xi, y = 0)$ are determined from (1.7) by the substitution $x = l\xi, y = 0$. Therefore, the solution of the problem can be calculated to any asymptotic accuracy by means of (1.4) and (1.7) and the recursion relations (1.5) and (1.9).

Limiting ourselves to the accuracy $O(\varepsilon^2)$ in determining the desired quantities, which will normally be sufficient for practical applications, we will have the solution (the prime denotes the derivative with respect to x):

For the isotropic layer $0 \leqslant y \leqslant h_1$

$$\sigma_{xy}^{(1)} = \sigma_{xy}^{(F)}, \quad \sigma_{yy}^{(1)} = \sigma_{yy}^{(F)} \tag{1.10}$$

$$\sigma_{xx}^{(1)} = \sigma_{xx}^{(F)} + \frac{h_2}{a_{11}^{(1)}}\left(A_{26}^{(2)}\bar{\sigma}_{yy}' + A_{22}^{(2)}\bar{\sigma}_{xy}' - \frac{1}{h_2}\bar{u}'\right)$$

$$u_x^{(1)} = u_x^{(F)} - \bar{u} + h_2(A_{26}^{(2)}\bar{\sigma}_{yy} + A_{22}^{(2)}\bar{\sigma}_{xy}) + y\bar{v}' - \left[A_{66}^{(2)}yh_2 + \frac{1}{2}h_2^2\left(\frac{a_{12}^{(2)}}{a_{11}^{(2)}}A_{22}^{(2)} - \frac{a_{16}^{(2)}}{a_{11}^{(2)}}A_{26}^{(2)} + A_{66}^{(2)}\right)\right]\bar{\sigma}_{yy}' - \left(\frac{1}{2}A_{26}^{(2)}yh_2 + \frac{a_{26}^{(2)} - a_{16}^{(2)}}{2a_{11}^{(2)}}A_{22}^{(2)}h_2^2\right)\bar{\sigma}_{xy}'$$

$$u_y^{(1)} = u_y^{(F)} - \bar{v} + h_2(A_{66}^{(2)}\bar{\sigma}_{yy} + A_{26}^{(2)}\bar{\sigma}_{xy}) + \frac{a_{12}^{(1)}}{a_{11}^{(1)}}yh_2(A_{26}^{(2)}\bar{\sigma}_{yy}' + A_{22}^{(2)}\bar{\sigma}_{xy}') + \frac{1}{2}h_2^2\left(\frac{a_{12}^{(2)}}{a_{11}^{(2)}}A_{22}^{(2)} - \frac{a_{26}^{(2)}}{a_{11}^{(2)}}A_{26}^{(2)} + A_{66}^{(2)}\right)\bar{\sigma}_{xy}'$$

$$a_{11}^{(1)} = a_{22}^{(1)} = \frac{1-\nu^2}{E}, \quad a_{12}^{(1)} = -\frac{\nu(1+\nu)}{E}$$

For the anisotropic layer $-h_2 \leqslant y \leqslant 0$

$$\sigma_{xy}^{(2)} = \bar{\sigma}_{xy} + y\left(\frac{a_{12}^{(2)}}{a_{11}^{(2)}}\bar{\sigma}_{yy}' + \frac{a_{26}^{(2)}}{a_{11}^{(2)}}\bar{\sigma}_{xy}'\right) \tag{1.11}$$

$$\sigma_{yy}^{(2)} = \bar{\sigma}_{yy} - y\bar{\sigma}_{xy}'$$

$$\sigma_{xx}^{(2)} = -\frac{1}{a_{11}^{(2)}}(a_{12}^{(2)}\bar{\sigma}_{yy} + a_{26}^{(2)}\bar{\sigma}_{xy}) - y\left[\frac{a_{12}^{(2)}a_{16}^{(2)}}{(a_{11}^{(2)})^2}\bar{\sigma}_{yy}' + \frac{a_{26}^{(2)}a_{16}^{(2)} - a_{12}^{(2)}a_{11}^{(2)}}{(a_{11}^{(2)})^2}\bar{\sigma}_{xy}'\right] + \frac{1}{a_{11}^{(2)}}(y + h_2)(A_{26}^{(2)}\bar{\sigma}_{yy}' + A_{22}^{(2)}\bar{\sigma}_{xy}')$$

$$u_x^{(2)} = (y + h_2)(A_{26}^{(2)}\bar{\sigma}_{yy} + A_{22}^{(2)}\bar{\sigma}_{xy}) + \frac{1}{2}(y^2 - h_2^2) \times \left[\left(\frac{a_{12}^{(2)}}{a_{11}^{(2)}}A_{22}^{(2)} + \frac{a_{16}^{(2)}}{a_{11}^{(2)}}A_{26}^{(2)} - A_{66}^{(2)}\right)\bar{\sigma}_{yy}' + \left(\frac{a_{16}^{(2)} + a_{26}^{(2)}}{a_{11}^{(2)}}A_{22}^{(2)} - 2A_{26}^{(2)}\right)\bar{\sigma}_{xy}'\right] + h_2(y + h_2)\left[\left(\frac{a_{16}^{(2)}}{a_{11}^{(2)}}A_{26}^{(2)} - A_{66}^{(2)}\right)\bar{\sigma}_{yy}' + \left(\frac{a_{16}^{(2)}}{a_{11}^{(2)}}A_{22}^{(2)} - A_{26}^{(2)}\right)\bar{\sigma}_{xy}'\right]$$

$$u_y^{(2)} = (y + h_2)(A_{66}^{(2)}\bar{\sigma}_{yy} + A_{26}^{(2)}\bar{\sigma}_{xy}) + y(y + h_2)\frac{a_{12}^{(2)}}{a_{11}^{(2)}}A_{26}^{(2)}\bar{\sigma}_{yy}' + \frac{1}{2}(y^2 - h_2^2)\left(\frac{a_{26}^{(2)}}{a_{11}^{(2)}}A_{26}^{(2)} + \frac{a_{12}^{(2)}}{a_{11}^{(2)}}A_{22}^{(2)} - A_{66}^{(2)}\right)\bar{\sigma}_{xy}' + h_2(y + h_2)\frac{a_{12}^{(2)}}{a_{11}^{(2)}}A_{22}^{(2)}\bar{\sigma}_{xy}'$$

Here

$$\bar{\sigma}_{xy} = \sigma_{xy}^{(F)}(y = 0) = \frac{2P}{\pi}\frac{xh_1^2}{(x^2 + h_1^2)^2} \tag{1.12}$$

$$\bar{\sigma}_{yy} = \sigma_{yy}^{(F)}(y = 0) = -\frac{2P}{\pi}\frac{h_1^3}{(x^2 + h_1^2)^2}$$

$$\bar{u} = u_x^{(F)}(y = 0) = -\frac{P(1+\nu)(1-2\nu)}{\pi E}\operatorname{arctg}\frac{x}{h_1} + \frac{P(1+\nu)}{\pi E}\frac{xh_1}{r^2(0,0)}$$

$$\bar{v} = u_y^{(F)}(y = 0) = \frac{2P(1-\nu^2)}{\pi E}\ln r(0,0) + \frac{P(1+\nu)}{\pi E}\frac{x^2}{r^2(0,0)}$$

2. If the load acting on the edge $y = h_1$ is given in the form of a piecewise-continuous function

$$\sigma_{yy}(x, y = h_1) = \sigma_{yy}^{+}(x) = \begin{cases} -p(x), & a \leqslant x \leqslant b \\ 0, & x \overline{\in} [a, b] \end{cases} \tag{2.1}$$
$$\sigma_{xy}(x, y = h_1) = \sigma_{xy}^{+}(x) = 0, \quad -l \leqslant x \leqslant l$$

the solution (1.10) and (1.11) retains its form but $\sigma_{xx}^{(F)}$, $\sigma_{jy}^{(F)}$, $u_j^{(F)}$ $(j = x, y)$ are given by (1.7), while $\bar{\sigma}_{jy} = \sigma_{jy}^{(F)}(x, 0)$, $\bar{u} = u_x^{(F)}(x, 0)$, $\bar{v} = u_y^{(F)}(x, 0)$.

We will present the solutions for certain cases often encountered in practice.

a) Let a uniformly distributed piecewise-continuous normal load (Fig.2) act at the edge of the strip $y = h_1$

$$p(x) = \begin{cases} \text{const}, & -a \leqslant x \leqslant a \\ 0, & |x| > a \end{cases} \tag{2.2}$$

We have, to an accuracy $O(\varepsilon^2)$ for the isotropic part of the strip $0 \leqslant y \leqslant h_1$

$$\sigma_{yy}^{(1)} = \sigma_{xy}^{(1)} + \frac{\sigma}{\pi} A(x, y) \tag{2.3}$$
$$\sigma_{xy}^{(1)} = \frac{\sigma}{\pi}\left[\frac{(x+a)(y-h_1)}{r^2(-a, y)} - \frac{(x-a)(y-h_1)}{r^2(a, y)}\right]$$
$$\sigma_{xx}^{(1)} = -\sigma_{xy}^{(1)} + \frac{\sigma}{\pi} A(x, y) + \frac{Eh_2}{1-\nu}\left(A_{26}^{(2)}\bar{\sigma}_{yy}' + A_{22}^{(2)}\bar{\sigma}_{xy}' - \frac{1}{h_2}\bar{u}'\right)$$
$$u_x^{(1)} = \frac{(1+\nu)(1-2\nu)\sigma}{\pi E}\left[(x+a)\operatorname{arctg}\frac{x+a}{y-h_1} - (x-a)\operatorname{arctg}\frac{x-a}{y-h_1}\right] +$$
$$\frac{2(1-\nu)\sigma}{\pi E}(y-h_1)\ln\frac{r(a, y)}{r(-a, y)} - \bar{u} + h_2\left(A_{26}^{(2)}\bar{\sigma}_{yy} + A_{22}^{(2)}\bar{\sigma}_{xy}\right) + y\bar{v}' -$$
$$\left[A_{66}^{(2)} y h_2 + \frac{1}{2} h_2^2\left(\frac{a_{12}^{(2)}}{a_{11}^{(2)}} A_{22}^{(2)} - \frac{a_{16}^{(2)}}{a_{11}^{(2)}} A_{23}^{(2)} + A_{66}^{(2)}\right)\right]\bar{\sigma}_{yy}' -$$
$$\left(\frac{1}{2} A_{26}^{(2)} y h_2 + \frac{a_{26}^{(2)} - a_{16}^{(2)}}{2a_{11}^{(2)}} A_{22}^{(2)} h_2^2\right)\bar{\sigma}_{xy}'$$
$$u_y^{(1)} = \frac{(1+\nu)(3-2\nu)\sigma}{\pi E}\left[2a - (y-h_1) A(x, y)\right] +$$
$$\frac{2(1-\nu^2)\sigma}{\pi E}\left[(x+a)\ln r(-a, y) - (x-a)\ln r(a, y)\right] -$$
$$\bar{v} + h_2\left(A_{66}^{(2)}\bar{\sigma}_{yy} + A_{26}^{(2)}\bar{\sigma}_{xy}\right) - \frac{\nu(1+\nu)}{1-\nu} A_{26}^{(2)} y h_2 \bar{\sigma}_{yy}' -$$
$$\left[\frac{\nu(1+\nu)}{1-\nu} A_{22}^{(2)} y h_2 - \frac{1}{2}\frac{a_{12}^{(2)}}{a_{11}^{(2)}} A_{22}^{(2)} h_2^2 + \frac{1}{2}\frac{a_{26}^{(2)}}{a_{11}^{(2)}} A_{26}^{(2)} h_2^2 - \frac{1}{2} A_{66}^{(2)} h_2^2\right]\bar{\sigma}_{xy}'$$
$$A(x, y) = \operatorname{arctg}\frac{2a(y-h_1)}{x^2 - a^2 + (y-h_1)^2}$$

while the quantities for the anisotropic part $-h_2 \leqslant y \leqslant 0$ are determined from (1.11) but

$$\bar{\sigma}_{yy} = \bar{\sigma}_{xy} + \frac{\sigma}{\pi} A(x, 0), \quad \bar{\sigma}_{xy} = \frac{\sigma}{\pi}\left[\frac{h_1(x-a)}{r^2(a, 0)} - \frac{h_1(x+a)}{r^2(-a, 0)}\right] \tag{2.4}$$
$$\bar{u} = \frac{(1+\nu)(1-2\nu)\sigma}{\pi E}\left[(x-a)\operatorname{arctg}\frac{x-a}{h_1} - (x+a)\operatorname{arctg}\frac{x+a}{h_1}\right] -$$
$$\frac{2(1-\nu^2)\sigma}{\pi E} h_1 \ln\frac{r(a, 0)}{r(-a, 0)}$$
$$\bar{v} = \frac{(1+\nu)(3-2\nu)\sigma}{\pi E}\left[2a + h_1 A(x, 0)\right] +$$
$$\frac{(1-\nu^2)\sigma}{\pi E}\left[(x+a)\ln r(-a, 0) - (x-a)\ln r(a, 0)\right]$$

should be taken in (1.11) and in (2.3).

b) A normal load varying linearly along the section $a \leqslant x \leqslant b$ (Fig.3)

$$p(x) = \begin{cases} kx, & a \leqslant x \leqslant b \\ 0, & x \overline{\in} [a, b] \end{cases} \tag{2.5}$$

acts on the edge $y = h_1$.

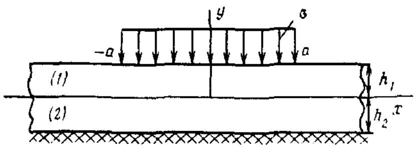

Fig.2

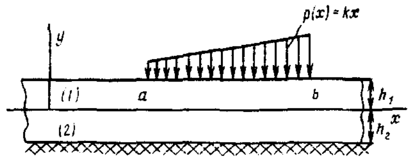

Fig.3

The states of stress and strain in the strip layers are determined, respectively, by means of (1.1) and (1.11) in which the following should be substituted:

$$\sigma_{xy}^{(F)} = \frac{k}{\pi}\left[\frac{b(y-h_1)^2}{r^2(b,y)} - \frac{a(y-h_1)^2}{r^2(a,y)}\right] + \frac{k}{\pi}(y-h_1)B$$

$$\sigma_{yy}^{(F)} = \frac{k}{\pi}(y-h_1)\left[\frac{x(b-x)-(y-h_1)^2}{r^2(b,y)} - \frac{x(a-x)-(y-h_1)^2}{r^2(a,y)}\right] - \frac{kx}{\pi}B$$

$$\sigma_{xx}^{(F)} = \frac{2k}{\pi}(y-h_1)\ln\frac{r(b,y)}{r(a,y)} - \frac{kx}{\pi}B + \frac{k(y-h_1)}{\pi}\left[\frac{x(x-b)+(y-h_1)^2}{r^2(b,y)} - \frac{x(x-a)+(y-h_1)^2}{r^2(a,y)}\right]$$

$$u_x^{(F)} = \frac{(1+\nu)(1-2\nu)k}{2\pi E}\left\{r^2(b,y)\operatorname{arctg}\frac{x-b}{y-h_1} - r^2(a,y)\operatorname{arctg}\frac{x-a}{y-h_1} + 2x(y-h_1)\ln\frac{r(b,y)}{r(a,y)} - 2x\left[(x-b)\operatorname{arctg}\frac{x-b}{y-h_1} - (x-a)\operatorname{arctg}\frac{x-a}{y-h_1}\right] + (a-b)(y-h_1)\right\} + \frac{(1+\nu)k}{\pi E}(y-h_1)\left[(y-h_1)B + x\ln\frac{r(b,y)}{r(a,y)} + b - a\right]$$

$$u_y^{(F)} = \frac{(1-\nu^2)k}{2\pi E}\left[r^2(b,y)\ln r(b,y) - r^2(a,y)\ln r(a,y) + 6x(b-a) + 2(a^2-b^2) - 2x(x-b)\ln r(b,y) + 2x(x-a)\ln r(a,y) + x(y-h_1)B\right] + \frac{(1+\nu)k}{2\pi E}\left[b^2 - a^2 + 2(y-h_1)^2\ln\frac{r(a,y)}{r(b,y)} - 2x(y-h_1)B\right]$$

$$B = \operatorname{arctg}\frac{(a-b)(y-h_1)}{(x-a)(x-b)+(y-h_1)^2}$$

$$\bar{u} = u_x^{(F)}(x,0),\quad \bar{v} = u_y^{(F)}(x,0),\quad \bar{\sigma}_{xy} = \sigma_{xy}^{(F)}(x,0),\quad \bar{\sigma}_{yy} = \sigma_{yy}^{(F)}(x,0)$$

The solutions for the other cases are also written down in an analogous manner.

All the solutions are valid starting with distances from the tranverse edges $x = \pm l$, that equal the zone of boundary-layer spread. This zone can be ascertained by the method described in /5/. To have a complete representation of the nature of the stress and displacement distributions near the edges $x = \pm l$, it is necessary to append a boundary-layer type solution to the solution mentioned.

Therefore, by combining the solution of the Flamant-Boussinesq problem with the asymptotic solution of the problem for a strip, the range of applications of asymptotic solutions can be extended considerably /4, 6, 8/. We also note that the first layer can also be anisotropic, in this case it is necessary to use the solution of the Flamant problem for an anisotropic half-space. The approach described can also be used in analogous problems for layers and plates.

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